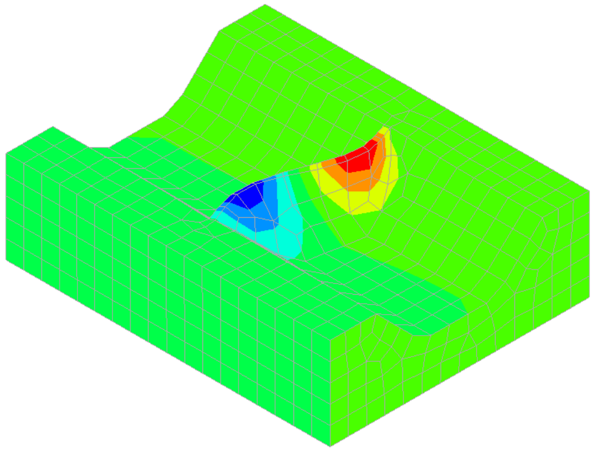
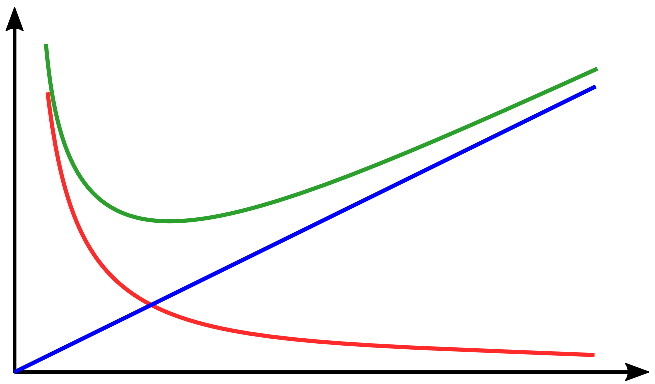
Tutorial

Rayleigh Damping Parameters of a Gravity Dam





Outline

1 Description

1.1 Model

In this tutorial we derive the Rayleigh damping parameters for the gravity dam presented in Figure 1 using DianaIE. The modeling requires the following steps:

- generation of the foundation and dam geometry
- assignment of the material properties
- application of the constraints
- assignment of the deadweight as the only load acting on the dam.

The geometry of the foundation and the dam and the corresponding material properties are the same as in the tutorial "*Response Spectrum Analysis of a Gravity Dam*". As shown in Table 1, all materials are linear elastic. Since we focus on the calculation of the Rayleigh damping parameters of the dam, the density of the foundation is assumed equal to zero. By assuming a massless foundation the wave speed becomes infinite and the excitation is applied at the dam at the same time as it is applied at the bottom of the model[1].

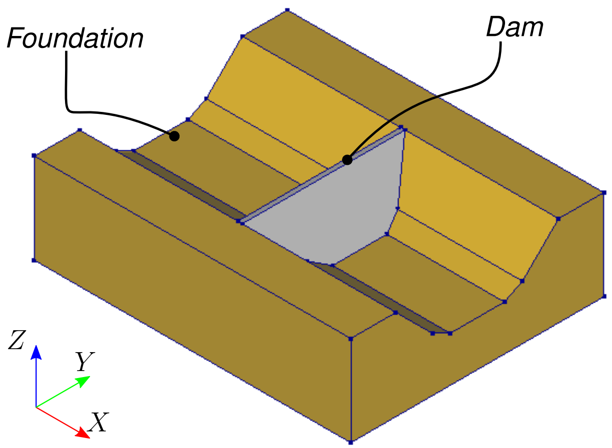


Figure 1: Geometric model of the dam

Foundation		
Young's modulus	8e+9	Pa
Poisson's ratio	0.2	
Density	0	kg/m^3
Dam		
Young's modulus	3.2e+10	Pa
Poisson's ratio	0.2	
Density	2400	kg/m^3

Table 1: Material properties

[1]The assumption of a massless foundation is for simplicity of this specific tutorial in which the user is shown how to perform an eigenvalue analysis to determine the Rayleigh damping parameters for the dam as preparation for a (transient) dynamic analysis. In reality the foundation is not linear elastic and certainly not massless. For more realistic (transient) dynamic analysis results it is better to use mass in the foundation and apply a non-linear, preferably cyclic, soil material. When the mass of the foundation is also included, the user can also apply absorbing boundary conditions for a (transient) dynamic analysis

1.2 Rayleigh Damping

When performing a transient analysis of structural models using Rayleigh damping, the damping matrix **C** is calculated as the linear combination of the mass matrix **M** and the stiffness matrix **K**:

$$\mathbf{C} = a\mathbf{M} + b\mathbf{K} \tag{1}$$

where a and b are the Rayleigh damping parameters related to the mass matrix and the stiffness matrix, respectively. As described in detail in Chopra (2007) [2], damping ratios ζ_n depend on the eigenfrequencies f_n of the structure and on the Rayleigh parameters a and b [Fig. 2]. More specifically, the damping ratio for the n^{th} eigenfrequency is:

$$\zeta_n = a\frac{1}{4\pi f_n} + b\pi f_n. \tag{2}$$

Typically, Equation (2) is expressed as a function of angular speed ω_n instead of the eigenfrequencies f_n. Thus, since $\omega_n = 2\pi f_n$, Equation (2) can be rewritten as:

$$\zeta_n = a\frac{1}{2\omega_n} + b\frac{\omega_n}{2}. \tag{3}$$

[2] Chopra, *Dynamics of structures – Theory and applications to earthquake engineering*, 2007

Therefore, the Rayleigh damping parameters a and b "*can be determined from specified damping ratios ζ_i and ζ_j for the i^{th} and j^{th} modes, respectively*"(Chopra 2007)[1]:

$$\frac{1}{2}\begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \zeta_i \\ \zeta_j \end{bmatrix}. \quad (4)$$

DianaIE calculates the Rayleigh damping parameters based on Equation (4) by choosing a set of damping ratios (ζ_i and ζ_j) and eigenfrequencies (f_i and f_j). The set of damping ratios are defined by the user while the eigenfrequencies f_i and f_j can be:

- explicitly specified,
- determined by providing the corresponding eigenmodes (e.g., *mode 1* and *mode 3*)
- determined based on the eigenfrequencies specified by a cumulative effective mass percentage in the global XYZ directions (e.g., 90%, 90% and 90%). In case the requested cumulative effective mass percentages cannot be reached for the calculated number of eigenfrequencies, the second frequency is the highest calculated eigenfrequency.

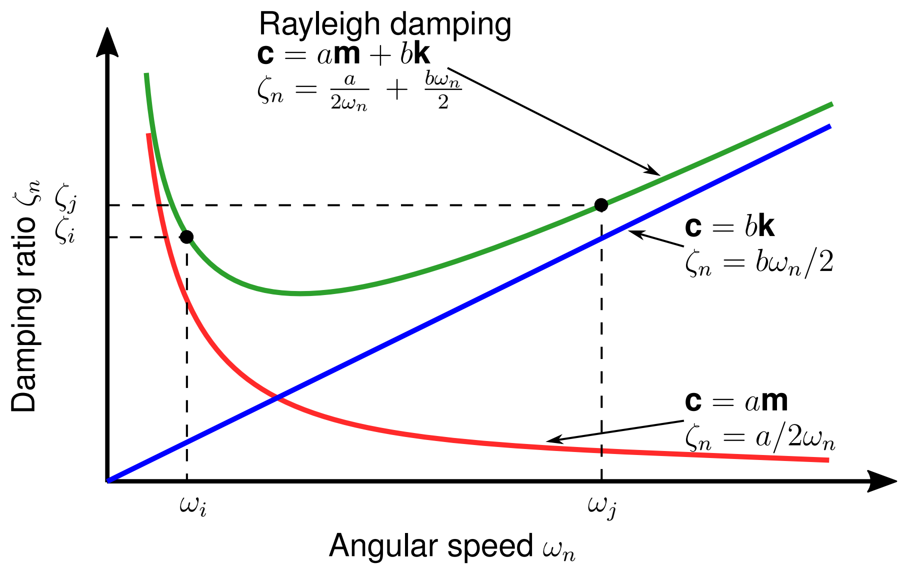


Figure 2: Variation of modal damping ratios ζ_n with the angular speed ω_n

In this tutorial we choose the eigenfrequencies f_i and f_j such that the cumulative effective mass percentage in all the three directions is higher or equal than 90%. Consequently, the first frequency (f_i) is the one associated to the first eigenmode (i.e., f_1) while the second corresponds to the smallest eigenfrequency associated to a cumulative effective mass percentage higher or equal than 90%.

As shown in Figure 3 for the first ten eigenmodes in the X direction (see the tutorial "*Response Spectrum Analysis of a Gravity Dam*"), the cumulative effective mass percentage at the seventh eigenmode is higher than 90%. The same holds for the cumulative effective mass percentage in the Y and Z directions as shown in Figure 4 and Figure 5, respectively.

In Figure 3 to Figure 5 we highlighted in red the eigenfrequency corresponding to the first mode and the first eigenfrequency associated to a cumulative effective mass percentage higher than 90%.

MODE	FREQUENCY	EFF.MASS TX	PERCENTAGE	CUM.PERCENT.
1	0.32146E+01	0.22358E+10	0.82238E+02	0.82238E+02
2	0.46589E+01	0.45582E+05	0.16766E-02	0.82240E+02
3	0.50870E+01	0.15346E+09	0.56445E+01	0.87884E+02
4	0.52876E+01	0.87955E+02	0.32352E-05	0.87884E+02
5	0.69385E+01	0.56161E+04	0.20657E-03	0.87884E+02
6	0.76034E+01	0.79240E+08	0.29146E+01	0.90799E+02
7	0.83398E+01	0.22000E+09	0.80919E+01	0.98891E+02
8	0.92600E+01	0.63419E+06	0.23327E-01	0.98914E+02
9	0.10617E+02	0.84353E+04	0.31027E-03	0.98914E+02
10	0.10840E+02	0.21681E+07	0.79746E-01	0.98994E+02

Figure 3: First ten eigenmodes in the X-direction and corresponding cumulative effective mass percentage

Therefore, the two angular speeds used for the calculation of the Rayleigh damping parameters are $\omega_1 = 2\pi \cdot f_1 = 2\pi \cdot 3.2146 = 19.57$ rad/s and $\omega_7 = 2\pi \cdot f_7 = 2\pi \cdot 7.6034 = 47.77$ rad/s.

MODE	FREQUENCY	EFF.MASS TY	PERCENTAGE	CUM.PERCENT.
1	0.32146E+01	0.18286E+05	0.67259E-03	0.67259E-03
2	0.46589E+01	0.26214E+10	0.96422E+02	0.96422E+02
3	0.50870E+01	0.83310E+05	0.30643E-02	0.96425E+02
4	0.52876E+01	0.20597E+08	0.75762E+00	0.97183E+02
5	0.69385E+01	0.45605E+08	0.16775E+01	0.98861E+02
6	0.76034E+01	0.13862E+03	0.50986E-05	0.98861E+02
7	0.83398E+01	0.11215E+05	0.41253E-03	0.98861E+02
8	0.92600E+01	0.65872E+04	0.24229E-03	0.98861E+02
9	0.10617E+02	0.99757E+02	0.36693E-05	0.98861E+02
10	0.10840E+02	0.38199E+04	0.14051E-03	0.98861E+02

Figure 4: First ten eigenmodes in the Y direction and corresponding cumulative effective mass percentage

MODE	FREQUENCY	EFF.MASS TZ	PERCENTAGE	CUM.PERCENT.
1	0.32146E+01	0.83985E+08	0.30891E+01	0.30891E+01
2	0.46589E+01	0.12131E+06	0.44620E-02	0.30936E+01
3	0.50870E+01	0.23536E+10	0.86570E+02	0.89663E+02
4	0.52876E+01	0.23854E+07	0.87740E-01	0.89751E+02
5	0.69385E+01	0.86632E+01	0.31865E-06	0.89751E+02
6	0.76034E+01	0.66481E+08	0.24453E+01	0.92197E+02
7	0.83398E+01	0.48316E+08	0.17772E+01	0.93974E+02
8	0.92600E+01	0.10500E+09	0.38621E+01	0.97836E+02
9	0.10617E+02	0.34870E+05	0.12826E-02	0.97837E+02
10	0.10840E+02	0.25216E+08	0.92750E+00	0.98765E+02

Figure 5: First ten eigenmodes in the Z direction and corresponding cumulative effective mass percentage

It is noteworthy that the choice of the frequencies f_i and f_j should also depend on the base acceleration used in the transient analysis.

In Figure 6 we show the accelerogram used in the tutorial "*Time History Analysis of a Gravity Dam*". From the corresponding Discrete Fourier Transform in Figure 7, obtained through the Fast Fourier algorithm, we observe that most of the energy content of the earthquake is between 0.5 and 4 Hz. Therefore, f_i and f_j should be chosen such that main frequencies of the earthquake are not filtered out.

In the present case, the eigenfrequency f_7=7.60 Hz complies with this requirements since it is higher than 4 Hz. On the other hand, f_1=3.21 Hz is too high as we do not want to filter out the frequencies of the input signal above 0.5 Hz. This can be achieved by using a smaller value of the damping ratio ζ_1 (by default, DianaIE assumes ζ_1 = 0.05 and ζ_7=0.05). Thus, we assume ζ_1=0.025 and Equation (4) becomes

$$\frac{1}{2}\begin{bmatrix}1/19.57 & 19.57\\ 1/47.77 & 47.77\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}0.025\\ 0.05\end{bmatrix} \quad\Rightarrow\quad \begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}0.2119\\ 0.0020\end{bmatrix} \tag{5}$$

We will later observe in Section 3.2 that the chosen set of eigenfrequencies and damping ratios will not filter out the relevant frequencies of the earthquake.

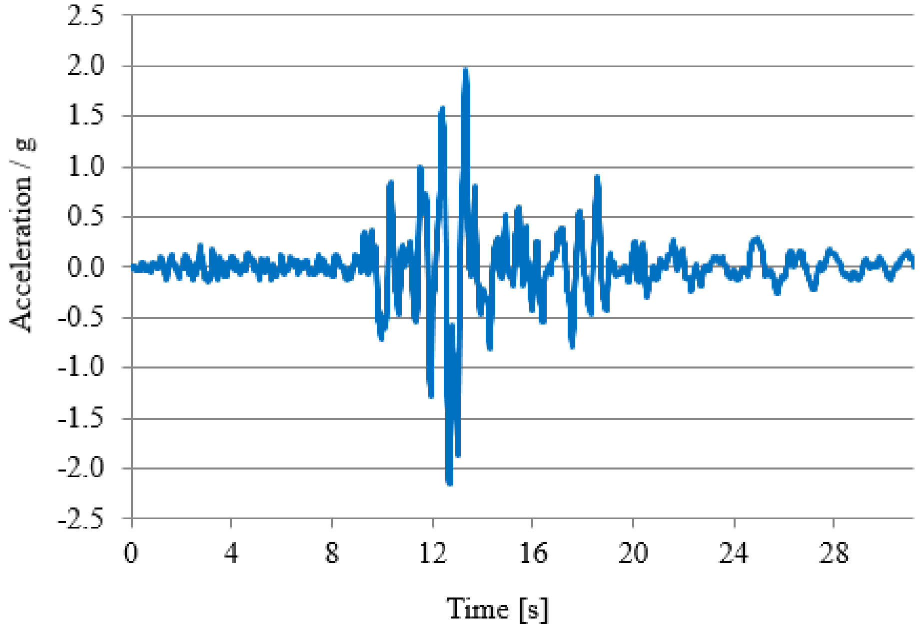


Figure 6: Accelerogram for base acceleration

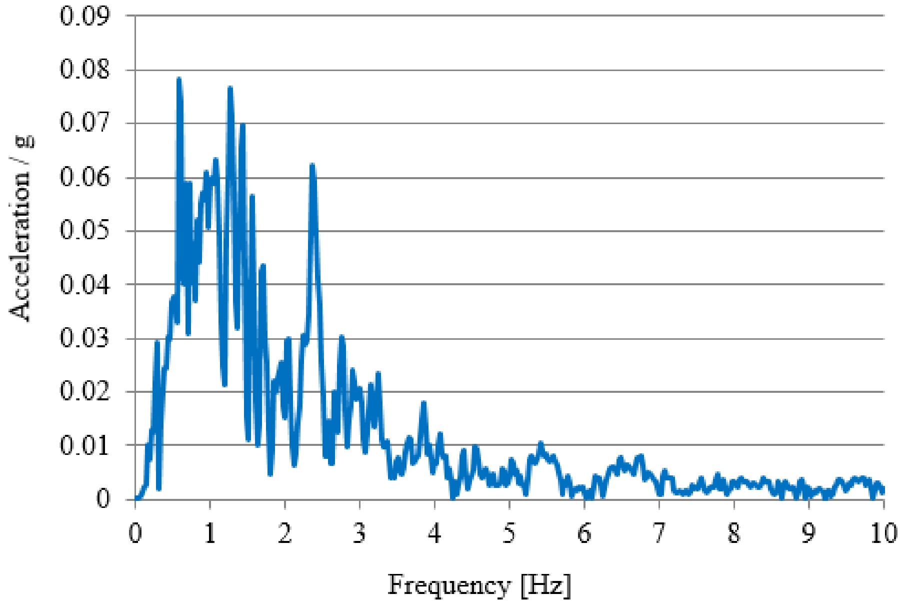


Figure 7: Discrete Fourier Transform of the accelerogram on the left

In this tutorial, the values of a and b in Equation (5) are calculated automatically in DianaIE.

2 Finite Element Model

The finite element model here employed is the same used in the tutorial "*Response Spectrum Analysis of a Gravity Dam*". The user is referred to the latter to create the geometry of the dam and the foundation and to assign the material properties, the constraints and the deadweight. For simplicity, compared to the aforementioned tutorial, we do not consider the reservoir, the fluid-structure interaction interface and the hydraulic pressure and fixed head potential.

Therefore, when starting the new project in DianaIE, we select to perform only a structural analysis as shown in Figure 8 (in the aforementioned tutorial structural and fluid-structure interaction analyses were performed).

The model used in the present tutorial is shown in Figure 9.

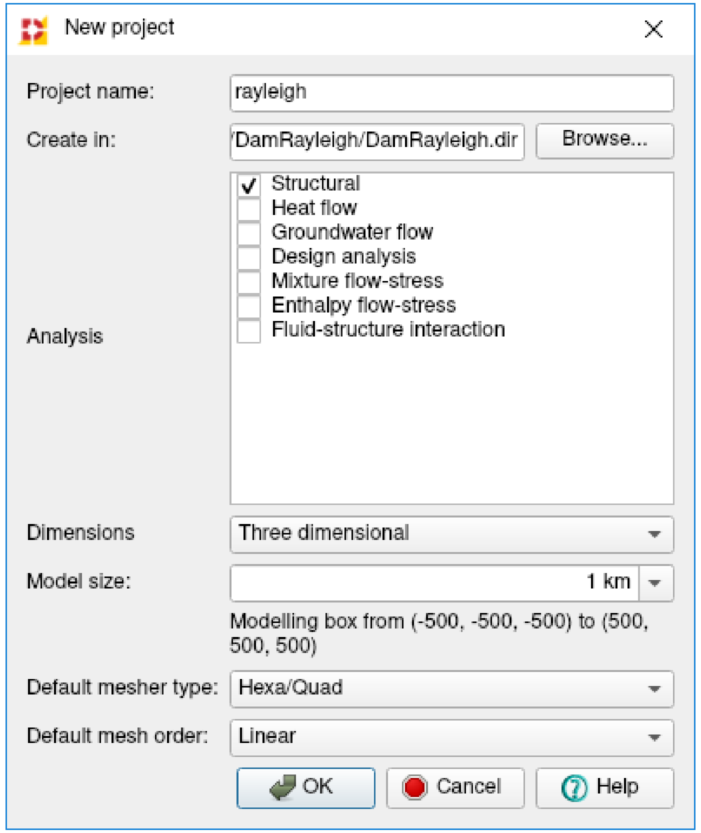


Figure 8: New project dialog

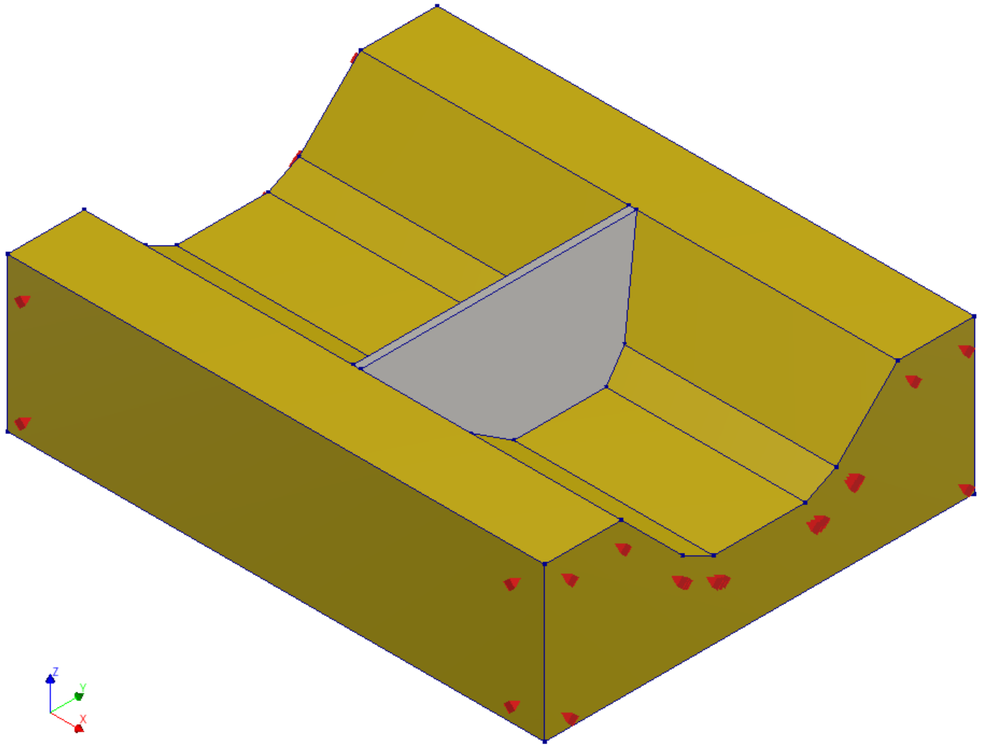


Figure 9: Geometry of the finite element model

2.1 Mesh

We set the mesh properties such that the elements size is equal to 40 m. Then, we generate the mesh.

DianaIE

Main menu → Geometry → Mesh → Mesh properties [Fig. 10]

Main menu → Geometry → Mesh → Generate mesh [Fig. 11]

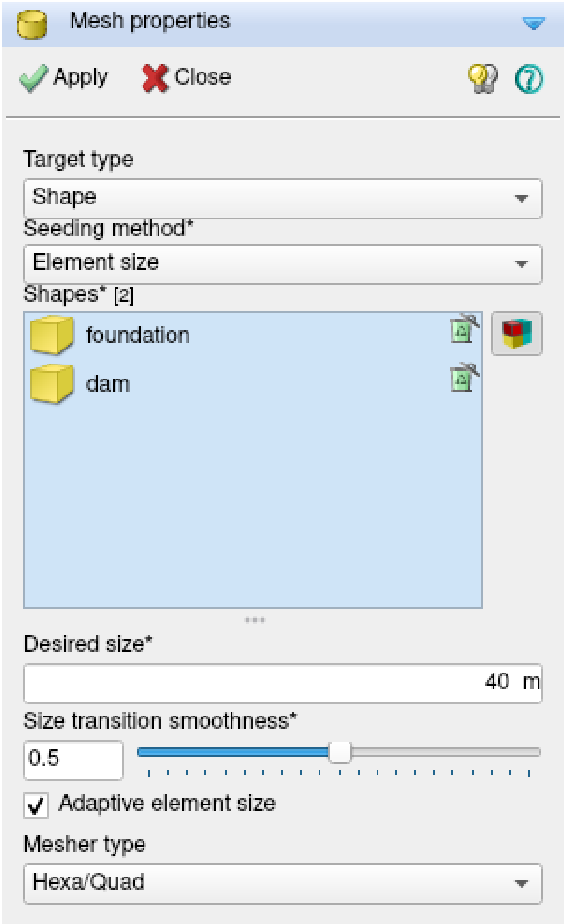


Figure 10: Mesh properties

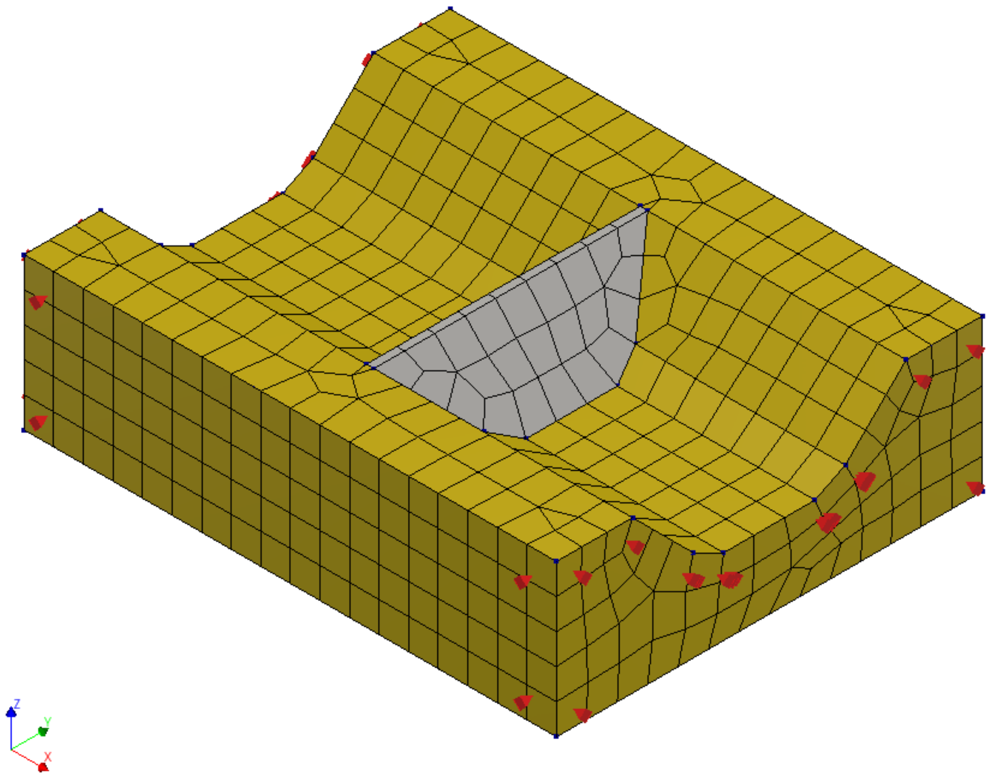


Figure 11: Finite element mesh

3 Structural Eigenvalue Analysis

3.1 Commands

We set up a structural eigenvalue analysis that, based on the calculation of the eigenfrequencies of the structure, allows to the determine the Rayleigh damping parameters of the dam. The deadweight load is considered for the geometric stress stiffness matrix in the eigenvalue analysis since it influences the eigenfrequencies of the dam.

DianaIE

Main menu → Analysis → Add analysis

Analysis browser → Analysis1 → Rename → Rayleigh Damping [Fig. 12]

Analysis browser → Rayleigh Damping → Add command → Structural eigenvalue [Fig. 13]

Analysis browser → Rayleigh Damping → Structural eigenvalue → Define eigenvalue type → Free vibration → Edit properties [Fig. 14] [Fig. 15]

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Figure 12: Analysis browser

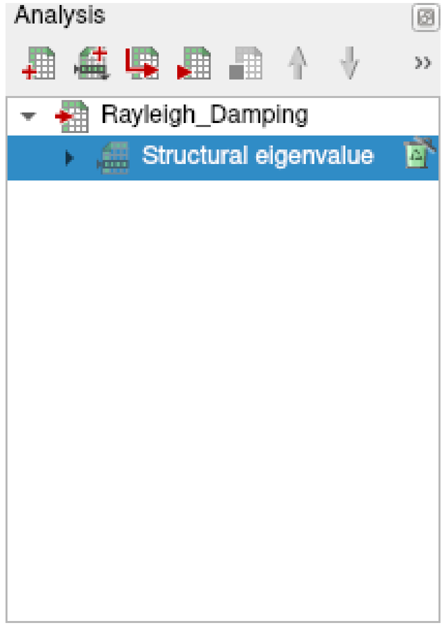

Figure 13: Analysis browser - add command

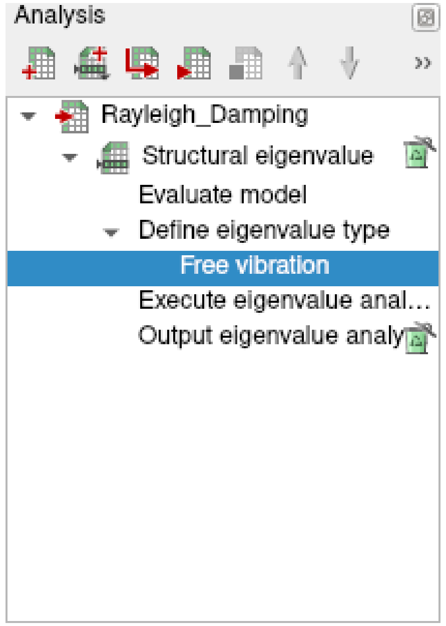

Figure 14: Analysis browser

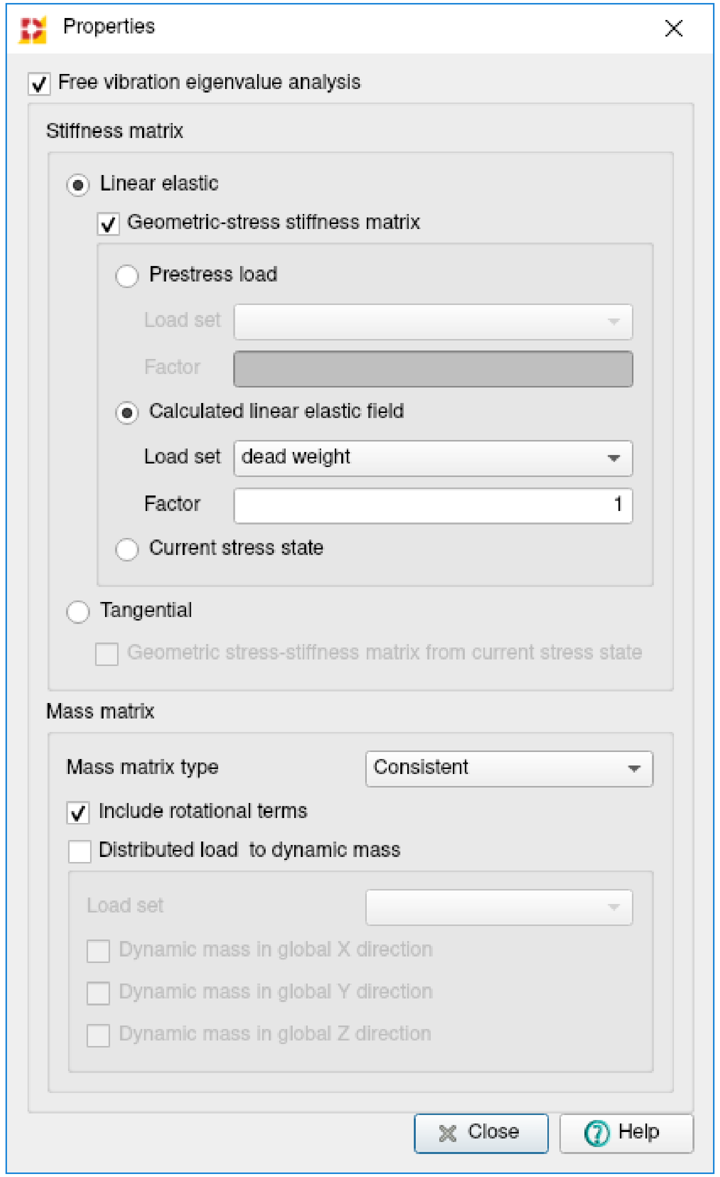

Figure 15: Edit free vibration properties

We choose to compute the first ten eigenmodes. As observed in the tutorial "*Response Spectrum Analysis of a Gravity Dam*", the first ten modes are enough to have a cumulative effective mass percentage above 90%.

DianaIE

Analysis browser → Rayleigh Damping → Structural eigenvalue → Execute eigenvalue analysis → Edit properties [Fig. 16] [Fig. 17]

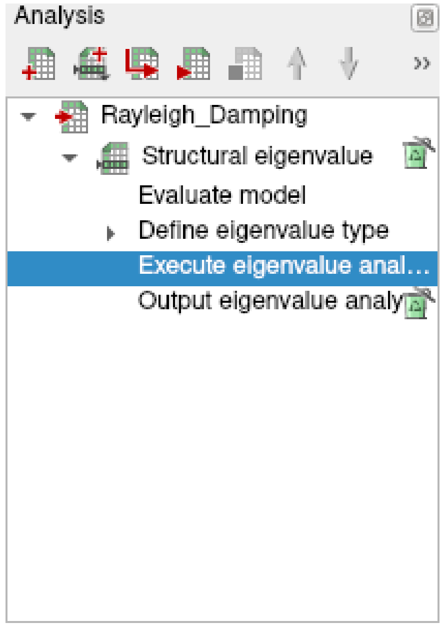

Figure 16: Analysis browser

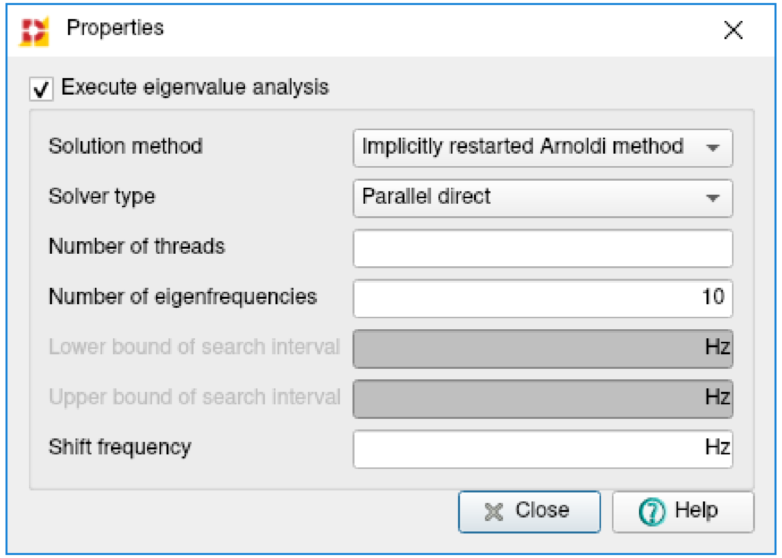

Figure 17: Execute eigenvalue analysis properties

We set up the eigenvalue analysis for the calculation of the Rayleigh damping parameters a and b. As mentioned in the previous section, we determine a and b such that the cumulative effective mass percentage is above 90% in all three global directions for the highest eigenfrequency used.

Then, we can run the analysis.

DianaIE

Analysis browser → Rayleigh Damping → Structural eigenvalue → Add... → Calculate Rayleigh parameters - effective mass [Fig. 18]
Analysis browser → Rayleigh Damping → Structural eigenvalue → Calculate Rayleigh parameters → Edit properties [Fig. 19] [Fig. 20]
Main menu → Analysis → Run selected analysis

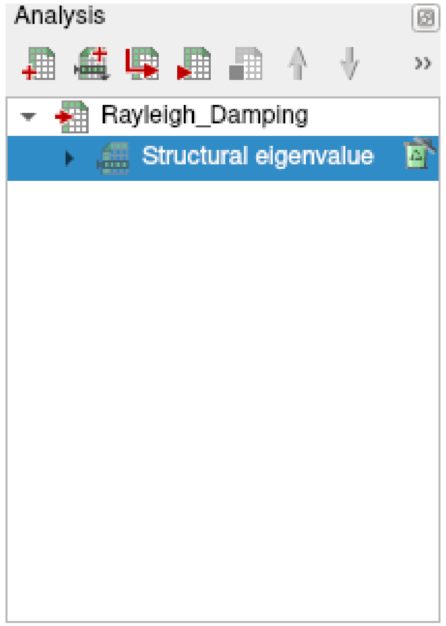

Figure 18: Analysis browser

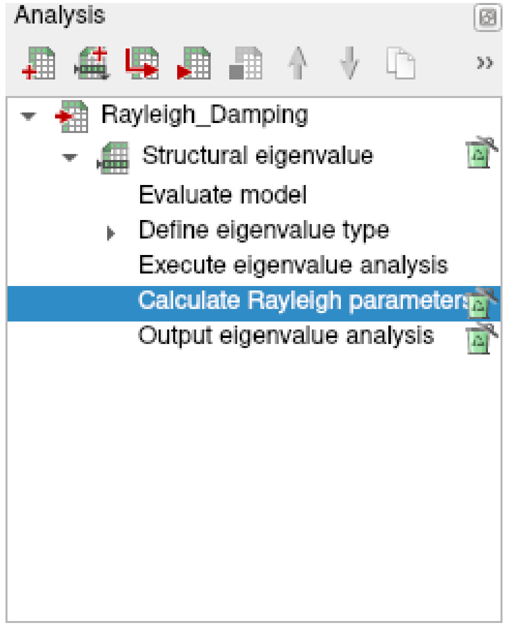

Figure 19: Analysis browser - add Rayleigh parameters calculation

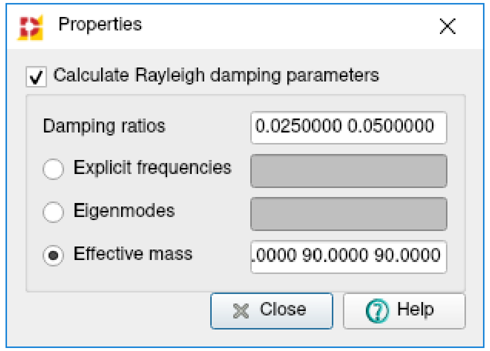

Figure 20: Edit calculation of Rayleigh parameters

3.2 Results: Rayleigh Damping Parameters

The calculated Rayleigh damping parameters are written in the standard DianaIE output file rayleigh_Rayleigh Damping.out as shown in Figure 21.

```
RAYLEIGH DAMPING COEFFICIENTS:
      FREQUENCY        DAMP.RATIO
     0.32146E+01      0.25000E-01
     0.76034E+01      0.50000E-01

 MASS FACTOR (a):               0.18990E+00
 STIFFNESS FACTOR (b):          0.20100E-02
```

Figure 21: Rayleigh damping parameters

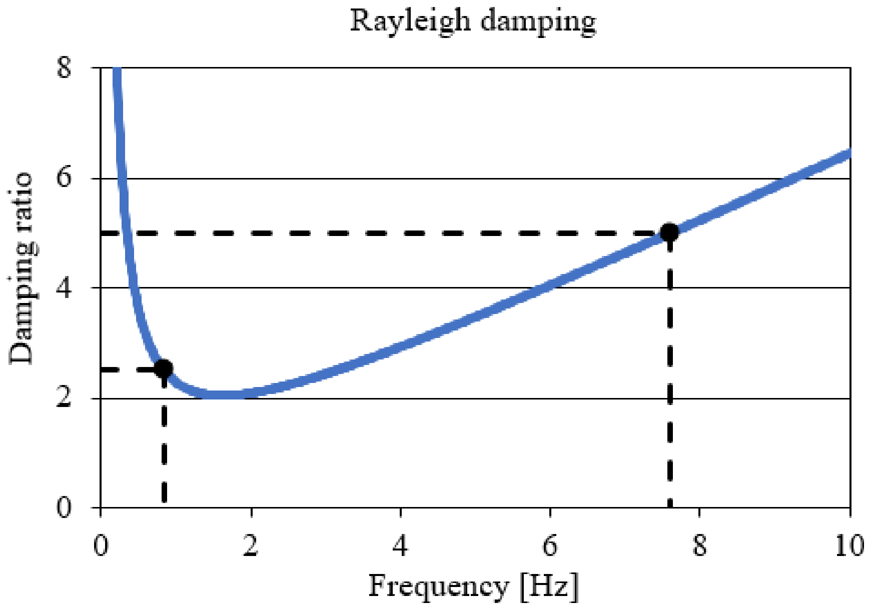

Figure 22: Damping ratio vs frequency

These values are in agreement with the hand calculations in Equation (5) performed earlier on slide 8.

Substituting the obtained values of a and b and using $\zeta_1 = 0.025$ and $\zeta_7 = 0.05$ in Equation (3) we can plot the curve in Figure 22. This graph shows that the earthquake frequencies between approximately 1 and 7.5 Hz are not overdamped for the calculated Rayleigh damping coefficients. Thus, the main frequencies of the input signal in Figure 7 are not being filtered out due to too much damping during the transient analysis.

Appendix A Additional Information

Folder: Tutorials/DamRayleigh

Number of elements $\approx$ 1500

Keywords:
ANALYS: eigen.
CONSTR: suppor.
ELEMEN: hx24l py15l solid te12l tp18l.
LOAD: weight.
MATERI: elasti isotro.
OPTION: direct.
POST: binary ndiana.
PRE: dianai.
RESULT: displa eigen total values.

References:

[1] A. K. Chopra. *Dynamics of structures – Theory and applications to earthquake engineering*. Prentice-Hall, 2007.

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